STATISTICAL CONSIDERATIONS

A1.1. Background

This appendix provides the statistical basis for presenting output data. This appendix relies heavily on the treatment given by Ziskin (1993).

Statistical methods can be very helpful in allowing for the determination of the most probable value or values of a quantity from a limited group of data. That is, given an experiment and the resulting data, it is possible to assess which value is the most likely to occur if the experiment is repeated.

However, a statistical evaluation cannot improve the accuracy of a measurement. The laws of probability utilized by statistics operate only on random errors and not on systematic errors. The systematic errors must be small compared to the random errors if the results of the statistical evaluation are to be meaningful.

The need to estimate population parameters from sample data stems from the fact that it is too expensive and/or not feasible to enumerate complete populations to obtain the required information. Statistical estimation procedures provide the means to obtain estimates of population parameters with desired precision.

Statistics is concerned with the theory and methodology for drawing inferences that extend beyond the particular set of data examined. Sample data are observed (or measured) in order to make inferences or decisions about the population from which the samples are drawn.

Two different types of estimates of population parameters are of interest: point estimates and interval estimates. A point estimate is a single number used as an estimate of an unknown population parameter such as the mean. Although any single point estimate is intended to be the true value, it will most likely deviate from it to some extent. It is thus necessary to have some measure of the error that might be involved in using this point estimate.

An interval estimate of a population parameter provides two values between which the (point estimate of the) parameter lies with a specified degree of confidence. There may be a high degree of confidence or very little confidence that the population parameter is included in the range of the interval estimate, so it is necessary to attach some sort of probabilistic statement to the interval. This is achieved by specifying confidence intervals and tolerance intervals. A 95% confidence interval specifies the range of values within which the mean (or some other population parameter) can be expected 95% of the time. A tolerance interval is used when the range of values in a population is of more interest than the average value. Statistical tolerance limits furnish limits between which we may confidently expect to find a given percentage of the individual values in a population.

Practical Implementation

<u>Example 1: Measurement of Ultrasonic Power</u>

In the way of an example of how the preceding principles can be applied in ultrasonic exposimetry, consider the measurement of acoustic power, P, using the radiation force technique.

The complete procedure requires the following steps:

1. Calibration of the radiation force balance
2. Performance of repeated measurements of power
3. Adjustment of data for correctable systematic errors
4. Calculation of random uncertainty (U_r)
5. Calculation of systematic uncertainty (U_s)
6. Calculation of total uncertainty (U_T)
7. Presentation of measured value in the form: $P = \bar{x} \pm U_T$

   where $\bar{x}$ is the sample mean

Calibration of Radiation Force Balance

Any calibration should be performed in a manner such that it is traceable to a national standards laboratory. A minimum of ten independent measurements of power output from the standard source should be performed at a minimum of two frequencies within the bandwidth of the ultrasonic systems for which power measurements are sought. Independence is defined operationally to mean that the entire set-up, measurement, and shut down procedures be utilized each and every time the measurement is made. This requires that between each measurement, a complete disassembly and start up of the measurement system be performed in accordance with the customary procedures at the start and end of the daily workday routine.

In accordance with the above guidelines, ten independent measurements are made of the NIST power standard source using a radiation force balance. The experimental procedure for setting up and using the reference source must be scrupulously followed according to the directions provided by NIST to ensure that the systematic uncertainty of the source will not exceed the specified value of ±4%.

Next, the reference power source is set to produce a specified output, say 1.000 watts at a specified frequency. Measurements obtained using the radiation force balance are shown in Table A-1. The mean value, 1.024 W, is larger than the presumed true value of 1.000 W. This constitutes a systematic error, but one that is correctable. Each power measurement will need to be multiplied by 1.000/1.024 = 0.9766 to compensate for the +2.4% bias. The corrected values are shown in Table A-1. From this point on, the original values are ignored, and consideration is given only to the corrected values. The mean value is now equal to the "true" value. The standard deviation (S_x) is given by:

A-1

$$S_x = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n - 1}} = 0.038\ W$$

The coefficient of variation, 0.0299/1.000, indicates a balance precision of 3.0%.

Performance of Repeated Measurements of Power

Having calibrated and characterized the measurement system, the radiation balance is now used to measure the power output of a transducer of interest. The frequency and operating controls are held constant. A number, say 5, repeated measurements are made. As described in the calibration procedure, a complete system disassembly and set-up is performed between measurements. The five measured values and their corrected values are listed in Table A-2, along with their mean and standard deviation.

Calculation of Random Uncertainty

The random uncertainty is computed using the following equation:

$$U_r = t_{.975} \frac{S_x}{\sqrt{n}} = 2.78 \frac{0.1093}{\sqrt{5}} = 0.136 \qquad \text{A-2}$$

where $t_{.975}$ is the value for a two-sided 95% confidence coefficient. The appropriate value of $t_{.975}$ for a sample size of 5 is obtained from Table A-3.

For comparative purposes, U_r may be divided by the mean, 2.133 W, to express the uncertainty in terms of per cent. Then,

$$U_r = \left(\frac{0.136}{2.133}\right) \cdot 100 = 6.37\% \qquad \text{A-3}$$

Calculation of Systematic Uncertainty

The total systematic uncertainty includes the systematic uncertainty specified by NIST (±4%) plus any non-corrected systematic error introduced at the measuring site. In this example, it is assumed that the only non-corrected systematic error at the measuring site is that which occurs in reading the balance scale. If the smallest scale division is 0.01 W, then a reasonable estimate of this systematic error is ± 0.002 W. The systematic error due to the reference source (as provided by NIST) is ± 4% times the mean value; that is ± (.04) x (2.133) = 0.085 W. Because there is no reason to suspect that any value would be more likely to occur than any other within their ranges of uncertainty, it is reasonable to assume that both of these systematic errors are uniformly distributed (rectangular distribution). Therefore, the overall systematic standard deviation is computed as

$$\sigma_s = \sqrt{\frac{a_1^2 + a_2^2}{3}} = \sqrt{\frac{(0.002)^2 + (0.085)^2}{3}} = 0.049\ W \qquad \text{A-4}$$

where a_1 and a_2 are the semiranges for the rectangular distributions. The total systematic uncertainty, U_s is given by

$$U_s = 1.96\ \sigma_s \tag{A-5}$$

for a 95% confidence level. However, if U_s is computed using equations A4 and A5 in the present example, the value of U_s (1.96 x 0.049 = 0.096 W) exceeds the sum of the individual semiranges (0.002 + 0.085 = 0.087), and is excessively pessimistic. Therefore, in this case, the total systematic uncertainty is computed as the sum of the dominant term (0.085) and the uncertainty associated with the remaining non-dominant terms (1.96 $\sqrt{[(0.002)^2/3]}$ = 0.0023) [Harris and Hinton, 1984]. Thus,

$$U_s = 0.085 + 0.002 = 0.087\ W \tag{A-6}$$

or, in terms of per cent,

$$U_s = \left(\frac{0.087}{2.133}\right) \cdot 100 = 4.1\ \% \tag{A-7}$$

Calculation of Total Uncertainty

The total uncertainty is the quadratic sum of the random and systematic uncertainties. Thus,

$$U_T = \sqrt{U_r^2 + U_s^2} = \sqrt{(0.136)^2 + (0.087)^2} = 0.161\ W \tag{A-8}$$

or in terms of percent

$$U_T = \left(\frac{0.161}{2.133}\right) \cdot 100 = 7.6\ \% \tag{A-9}$$

Presentation of Power Measurement

The ultrasonic power output of the transducer should be specified in the format: $P = \bar{x} \pm U_T$. That is,

A-10

$$\text{Ultrasonic Power} = 2.13 \pm 0.16 \text{ W} \qquad (95\% \text{ C.I.})$$

Example 2: Measurement of the Power Output of a Transducer Model

A frequent requirement for ultrasound equipment manufacturers is to specify the acoustic output of a physical quantity for an entire transducer model in production. In this case, the pertinent parent population is the set of all existing transducers of this model and all such transducers that might be produced in the future. From this presumed infinite population, a sample of n transducers is drawn, and r repeated measurements are obtained on each unit.

Table A-4 shows the power measurements obtained on a sample of 4 transducers ($n = 4$). Six repeated measurements are made on each transducer ($r = 6$). All of the procedures enumerated in the previous example are assumed to have been performed for the present measurements.

The basic concepts to be applied in this case are discussed in the following section on repeated measurements, in which all measurements are assumed to be independent and normally distributed.

Repeated Measurements

When multiple measurements are made on each transducer, the overall variability contains contributions from both the transducer variability and the inherent variability of the measurement system. The standard deviations corresponding to each of these two sources of error, as well as the overall standard deviation, can be estimated from the data in the following manner. For r repeated measurements on n transducers, calculate the mean, m_i, and the standard deviation, S_i, for each transducer. The overall standard deviation, S_x, is the standard deviation computed from the n transducer mean values. It is related to the standard deviation due to inter-transducer variability, S_{tx}, and the standard deviation due to measurement error, S_{meas}, by

A-11

$$S_x = \sqrt{S_{tx}^2 + \frac{S_{meas}^2}{r}}$$

S_{meas} is calculated using the transducer standard deviations in the equation:

A-12

$$S_{meas} = \sqrt{\frac{\sum_{i=1}^{n} S_i^2}{n}}$$

and S_{tx} is then

A-13

$$S_{tx} = \sqrt{S_x^2 - \frac{S_{meas}^2}{r}}$$

The overall mean, $\bar{x}$, is the mean of the n transducer mean values. It and the overall standard deviation, S_x, are used in the equations for computing the confidence and tolerance intervals. The number of degrees of freedom is n-1 in both computations, since S_x is computed from n values.

It should be noted that by measuring more transducers, the confidence interval as well as the measurement error will decrease in size proportional to $\sqrt{n}$, but the tolerance interval will approach a constant width representing the inherent inter-transducer variability.

Returning to the example in Table A-4, the mean and standard deviation are computed for the 6 values for each transducer. The overall mean, $\bar{x}$, is the mean of the individual means. That is,

A-14

$$\bar{x} = \frac{72 + 85 + 76 + 62}{4} = 73.75\ mW$$

The overall standard deviation, S_x, is the standard deviation of the 4 transducer values. That is,

A-15

$$S_x = \sqrt{\frac{\sum_{i=1}^{n}(m_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum_{i=1}^{n} m_i^2 - n\cdot\bar{x}^2}{n-1}}$$

$$= \sqrt{\frac{(72)^2 + (85)^2 + (76)^2 + (62)^2 - 4\cdot(73.75)^2}{4-1}}$$

$$= 9.54\ mW$$

The variability due to the measurement technique is quantified by the measurement standard deviation, S_{meas}. That is,

A-16

$$S_{meas} = \sqrt{\sum_{i=1}^{n}\frac{S_i^2}{n}} = \sqrt{\frac{(13.34)^2 + (7.77)^2 + (9.88)^2 + (8.22)^2}{4}}$$

$$= 10.04\ mW$$

The standard deviation arising from the inter-transducer variability is computed as

A-17

$$S_{tx} = \sqrt{S_x^2 - \frac{S_{meas}^2}{r}} = \sqrt{(9.54)^2 - \frac{(10.04)^2}{6}}$$

$$= 8.61\ mW$$

The random uncertainty, U_r, is

A-18

$$U_r = t_{.975}\frac{S_x}{\sqrt{n}} = 3.18\frac{9.54}{\sqrt{4}}$$
$$= 15.17\ mW$$

where 3.18 is the value for t for 3 degrees of freedom at the 95% confidence level.

For this example, the systematic uncertainty, U_s, will be assumed to be approximately the same as in the previous example. That is, $U_s = 4.5\%$, or in absolute terms,

A-19

$$U_s = 4.5\% \cdot \bar{x} = (0.045)(73.75) = 10.69\ mW$$

The total uncertainty, U_T, is then

A-20

$$U_T = \sqrt{U_r^2 + U_s^2} = \sqrt{(15.17)^2 + (10.69)^2}$$
$$= 18.56\ mW$$

The power output for this transducer model should be reported as:

A-21

$$\text{Power} = \bar{x} \pm U_T = 73.75 \pm 18.56\ \text{mW}$$
$$= 74 \pm 19\ \text{mW} \qquad (95\%\ \text{C.I.})$$

where the final value has been rounded off to the precision appropriate for the measured values.

Equation A-21 is a statement about the average output for this transducer model. It is also possible to provide an upper 95% tolerance limit for some proportion, say 99%, of the transducer units in this model. This is computed with the appropriate one-sided tolerance coefficient (Table A-5) to obtain a tolerance limit pertaining to the random error.

A-22

$$x \le \bar{x} + K_{.99}S_x = 73.75 + (7.04)(9.54)$$
$$\le 73.75 + 67.16 = 140.9\ mW$$

To obtain an upper tolerance limit incorporating systematic uncertainty, the systematic uncertainty will have to be quadratically added to the value of $K_{.99}S_x$. The correct upper tolerance level is then given by,

A-23

$$
\begin{aligned}
x &\le 73.75 + \sqrt{(K_{.99} S_x)^2 + U_s^2} \\
&\le 73.75 + \sqrt{(67.16)^2 + (10.69)^2} \\
&\le 142\ mW
\end{aligned}
$$

Thus, we can be 95% confident that 99% of the transducers will have power levels below 142 mW.

This example shows the value of making repeated measurements. Equations A-16 and A-17 show that the variability due to the measurement technique (S_{meas} = 10.04 mW) was greater than the variability amongst the transducers (S_{tx} = 8.61 mW). By making the repeated measurements, the measurement component of the overall variation was reduced by a factor of ~16. The resulting overall standard deviation (S_x = 9.54 mW) was made less than that of the measurement system standard deviation (S_{meas} = 10.04 mW).

Example 3: Measurement Of The Power Output Of An Ultrasound Scanner

An ultrasound scanner consists of a console plus one or more transducers. A frequent requirement for ultrasound equipment manufacturers is to specify the acoustic output for an entire production line of a console-transducer combination model. In this case the pertinent parent population is the set of all existing console-transducer combinations of this model and all such scanners that might be produced in the future. From this presumed infinite population, a sample of p transducers and q consoles are drawn. Each of the p transducers is tested with each of the q consoles for r repeated acoustic power output measurements. This arrangement is called a two-way crossed analysis of variance with repeated measurements.

In this analysis it is assumed that the consoles and transducers are independent and that all repeated measurements are independent. It is also assumed that all preliminary steps, such as correcting for systematic errors, have been performed as described previously in example 1.

Table A-6 shows the set-up for analyzing r repeated measurements for each of the pq console-transducer combinations. For the sake of brevity, the r individual power measurements for each combination (x_{ijk}) are not shown, only the mean (m_{ij}) and standard deviation (s_{ij}) are given.

The mean for each transducer i is given by:

A-24

$$m_{i\cdot} = \frac{1}{q} \sum_{j=1}^{q} m_{ij}$$

and the value is placed in the rightmost column at the appropriate level. Similarly, the mean for each console j is given by:

A-25

$$m_{\cdot j} = \frac{1}{p}\sum_{i=1}^{p} m_{ij}$$

The overall mean $\overline{\overline{m}}$ is given by either of the following equivalent expressions:

A-26

$$\overline{\overline{m}} = \frac{1}{pq}\sum_{i=1}^{p}\sum_{j=1}^{q} m_{ij} = \frac{1}{p}\sum_{i=1}^{p} m_{i\cdot} = \frac{1}{q}\sum_{j=1}^{q} m_{\cdot j}$$

Standard deviations are handled in a manner similar to that of means. The standard deviation of the r repeated measurements in the ij[th] cell (s_{ij}), is given by:

A-27

$$s_{ij} = \sqrt{\frac{\sum_{k=1}^{r}\left(x_{ijk} - m_{ij}\right)^2}{r - 1}}$$

The total standard deviation relative to the transducer means is:

A-28

$$s_{i\cdot} = \sqrt{\frac{\sum_{i=1}^{p}\left(m_{i\cdot} - \overline{\overline{m}}\right)^2}{p - 1}}$$

and the total standard deviation of the console means is given by:

A-29

$$s_{\cdot j} = \sqrt{\frac{\sum_{j=1}^{q}\left(m_{\cdot j} - \overline{\overline{m}}\right)^2}{q - 1}}$$

The above values are utilized in the calculation of the measurement standard deviation (S_{meas}), the transducer standard deviation, (S_{trans}), the console standard deviation ($S_{console}$), and the overall standard deviation (S_x). The measurement standard deviation (S_{meas}) is the square root of the average variance of each cell, and represents the intrinsic variability of the measurement process, per se. It is given by

A-30

$$S_{meas} = \sqrt{\frac{1}{pq}\sum_{i=1}^{p}\sum_{j=1}^{q} S_{ij}^{2}}$$

The variability due just to transducers is expressed in S_{trans} which is given by:

A-31

$$S_{trans} = \sqrt{S_{i\cdot}^{2} - \frac{1}{rq} S_{meas}^{2}}$$

The total variance about the transducer means ($S_{i\cdot}^{2}$) contains two components: one due to the measurement process (S_{meas}^{2}) and one due to variability of just the transducers (S_{trans}^{2}). S_{trans}^{2} is given by:

A-32

$$S_{trans}^{2} = S_{i\cdot}^{2} - \frac{1}{rq} S_{meas}^{2}$$

Similarly, the variability of just the consoles is given by:

A-33

$$S_{cons}^{2} = S_{\cdot j}^{2} - \frac{1}{rp} S_{meas}^{2}$$

The total variability of the measurements is the sum of the three sources of variability; that is

A-34

$$S_{x}^{2} = S_{trans}^{2} + S_{cons}^{2} + S_{meas}^{2}$$

S_x is the estimate of the standard deviation of the parent population of all output power measurement values. However, the standard deviation of the measurement mean of a sample containing r repeated measurements, p transducers and q consoles is

A-35

$$S_{\bar{x}} = \sqrt{\frac{S_{i\cdot}^{2}}{p} + \frac{S_{\cdot j}^{2}}{q} - \frac{S_{meas}^{2}}{rpq}}$$

S_x has DF = rpq - 1 degrees of freedom, and ($S_{\bar{x}}$) has DF = pq - 1 degrees of freedom.

The overall mean ($\bar{\bar{m}}$), the standard deviation of the mean ($S_{\bar{x}}$), and its degrees of freedom are used to compute the confidence interval. The overall mean ($\overline{\overline{m}}$), the standard deviation of all x_{ijk} values (S_x), and its degrees of freedom are used to calculate the tolerance interval.

Table A-7 shows the results of 6 repeated power measurements for each of the 12 console-transducer combinations. Each cell shows the mean and standard deviation of the six measurements. The transducer means are calculated using equation A-24. That is,

A-36

$$m_{1.} = \frac{1}{q}\sum_{j=1}^{q} m_{1j} = (72 + 62 + 64 + 68) / 4 = 66.50$$

$$m_{2.} = \frac{1}{q}\sum_{j=1}^{q} m_{2j} = (75 + 57 + 76 + 61) / 4 = 67.25$$

$$m_{3.} = \frac{1}{q}\sum_{j=1}^{q} m_{3j} = (45 + 52 + 49 + 51) / 4 = 49.25$$

The console means are calculated using equation A-25.

A-37

$$m_{.1} = \frac{1}{p}\sum_{i=1}^{p} m_{i1} = (72 + 75 + 45) / 3 = 64.00$$

$$m_{.2} = \frac{1}{p}\sum_{i=1}^{p} m_{i2} = (62 + 57 + 52) / 3 = 57.00$$

$$m_{.3} = \frac{1}{p}\sum_{i=1}^{p} m_{i3} = (64 + 76 + 49) / 3 = 63.00$$

$$m_{.4} = \frac{1}{p}\sum_{i=1}^{p} m_{i4} = (68 + 61 + 51) / 3 = 60.00$$

The overall mean is equal to the average of the transducer means (and also the average of the console means). That is

A-38

$$\bar{\bar{m}} = \frac{1}{p}\sum_{i=1}^{p} m_{i.} = (66.51 \;+\; 67.25 \;+\; 49.25) \;/\; 3 \;=\; 61.00$$

The total transducer standard deviation is given by equation A-28. However, for computational purposes, it is also given by:

A-39

$$s_{i.} \;=\; \sqrt{\frac{\sum_{i=1}^{p} m_{i.}^2 \;-\; p\bar{\bar{m}}^2}{p \;-\; 1}} \;=\; \sqrt{\frac{66.50^2 \;+\; 67.25^2 \;+\; 49.25^2 \;-\; 3\cdot(61.00)^2}{3 \;-\; 1}}$$

$$= \;10.18$$

Similarly, the total console standard deviation is:

A-40

$$s_{.j} \;=\; \sqrt{\frac{\sum_{j=1}^{q} m_{.j}^2 \;-\; q\,\bar{\bar{m}}^2}{q \;-\; 1}} \;=\; \sqrt{\frac{64.00^2 \;+\; 57.00^2 \;+\; 63.00^2 \;+\; 60.00^2 - \;4\cdot(61.00)^2}{4 \;-\; 1}}$$

$$= \;3.16$$

The measurement standard deviation (S_{meas}) is given by:

A-41

$$S_{meas} \;=\; \sqrt{\frac{1}{pq}\sum_{i=1}^{p}\sum_{j=1}^{q} S_{ij}^2}$$

$$= \;\sqrt{\frac{1}{3\cdot 4}(13.34^2 \;+\; 7.77^2 \;+\; 5.73^2 \;+\; 8.22^2 \;+\; 4.69^2 \;+\; \ldots. \;+\; 4.73^2}$$

$$= \;7.77$$

The standard deviation due to just transducer variability (S_{trans}) is given by equation A-31:

A-42

$$s_{trans} = \sqrt{s_{i.}^2 - \frac{1}{rq} s_{meas}^2} = \sqrt{10.18^2 - \frac{7.77^2}{6 \cdot 4}} = 10.06$$

The standard deviation due to just the console variability is

A-43:

$$s_{cons} = \sqrt{s_{.j}^2 - \frac{1}{rp} s_{meas}^2} = \sqrt{3.16^2 - \frac{7.77^2}{6 \cdot 3}} = 2.58$$

The standard deviation of all the measurements (S_x) is equal to

A-44

$$s_x = \sqrt{s_{trans}^2 + s_{cons}^2 + s_{meas}^2} = \sqrt{10.06^2 + 2.58^2 + 7.77^2}$$

$$= 12.97$$

The standard deviation of the measurement mean, sometimes called the standard error of the mean is given by:

A-45

$$s_{\bar{x}} = \sqrt{\frac{s_{trans}^2}{p} + \frac{s_{cons}^2}{q} + \frac{s_{meas}^2}{rpq}} = \sqrt{\frac{10.06^2}{3} + \frac{2.58^2}{4} + \frac{7.77^2}{6 \cdot 3 \cdot 4}}$$

$$= 6.02$$

The random uncertainty is given by:

A-46

$$u_r = t_{.975} \cdot s_{\bar{x}} = (2.20)\,(6.02) = 13.24\,,$$

where 2.20 is the value of t for 11 degrees of freedom at the 95% confidence level.

For this example the systematic uncertainty will be assumed to be the same as in the two previous examples. That is, $u_s = 4.5\%$ or, in absolute terms,

A-47

$$u_s = 4.5\% \,\overline{\overline{m}} = (0.045)\,(61.00) = 2.75$$

The total uncertainty, u_T, is:

A-48

$$u_T = \sqrt{u_r^2 + u_s^2} = \sqrt{13.24^2 + 2.75^2} = 13.52$$

Thus, the power output for this production line of ultrasound scanners should be reported as

A-49

$$\begin{aligned} Power &= m \pm u_T = 61.00 \pm 13.52 \\ &= 61 \pm 14 \text{ mW} \qquad (95\% \text{ C.I.}), \end{aligned}$$

where the final value has been rounded off to the precision appropriate for the measured quantities.

The upper 95% tolerance limit is given by the product of the overall standard deviation of all measurements and the appropriate value of K selected from table A-5 with the associated degrees of freedom. For 99% of measurements to be below the upper 95% tolerance limit, use $K_{.99}$ with DF = rpq - 1 = (6 · 3 · 4) - 1 = 71. Since a value for $K_{.99}$ for 71 degrees of freedom is not given explicitly in Table A-5, it must be interpolated as follows:

A-50

$$\begin{aligned} K_{.99} &= K_{.99}(99) + \left(\frac{99 - 71}{99 - 49} \right) \left[K_{.99}(49) - K_{.99}(99) \right] \\ &= 2.68 + 0.56 \ (2.86 - 2.68) = 2.78 \end{aligned}$$

The upper tolerance limit is thus expressed as

A-51

$$\begin{aligned} Power &\leq \overline{\overline{m}} + \sqrt{(K_{.99} \cdot s_x)^2 + u_s^2} \\ &\leq 61.00 + \sqrt{(2.78 \cdot 12.97)^2 + 2.75^2} = 97.2 \end{aligned}$$

Thus,

A-52

$$\text{Power} \leq 98 \text{ mW}$$

where the upper limit has been rounded upward to the precision appropriate for the measured quantities. Therefore, we can be 95% confident that 99% of all console-transducer combinations of the test model will have acoustic output powers less than 98 milliwatts.

TABLE A-1
CALIBRATION OF RADIATION FORCE BALANCE

	Measured Values (watts)		Corrected Values (watts)
	1.03		1.006
	1.00		0.977
	1.03		1.006
	1.05		1.025
	1.06		1.035
	0.98		0.957
	1.03		1.006
	1.07		1.045
	1.00		0.977
	0.99		0.967
Mean	1.024	**Mean**	1.000
Std. Dev.	0.0306	**Std. Dev.**	0.0299

TABLE A-2

MEASUREMENT OF ULTRASONIC POWER

	Measured Values (watts)		Corrected Values (watts)
	2.12		2.070
	2.34		2.285
	2.07		2.022
	2.26		2.207
	2.13		2.080
Mean	2.184	**Mean**	2.1328
Std. Dev.	0.1119	**Std. Dev.**	0.1093

TABLE A-3

95% and 99% Confidence Coefficients (Two-Sided)

Sample Size (n)	Degrees of Freedom (DF)	95% Confidence Coefficient ($t_{.975}$)	99% Confidence Coefficient ($t_{.995}$)
2	1	12.71	63.66
3	2	4.30	9.93
4	3	3.18	5.84
5	4	2.78	4.60
6	5	2.57	4.03
7	6	2.45	3.71
8	7	2.36	3.50
9	8	2.31	3.36
10	9	2.26	3.25
11	10	2.23	3.17
12	11	2.20	3.11
13	12	2.18	3.06
14	13	2.16	3.01
15	14	2.15	2.98
16	15	2.13	2.95
17	16	2.12	2.92
18	17	2.11	2.90
19	18	2.10	2.88
20	19	2.09	2.86
21	20	2.09	2.85
22	21	2.08	2.83
23	22	2.07	2.82
24	23	2.07	2.81
25	24	2.06	2.80
50	49	2.01	2.68
100	99	1.99	2.63
∞	∞	1.96	2.58

TABLE A-4

POWER MEASUREMENTS ON FOUR TRANSDUCERS
(Values are expressed in milliwatts)

	TRANSDUCER			
	A	**B**	**C**	**D**
	64	78	75	55
	72	91	93	66
	68	97	78	49
	77	82	71	64
	56	85	63	70
	95	77	76	68
MEAN	72	85	76	62
STD. DEV.	13.34	7.77	9.88	8.22

Overall Mean (the mean of all 24 values) = 73.75 mW
Overall Standard Deviation = 9.54 mW

Power = 74 ± 19 mW (95% C.I.)

TABLE A-5

95% Tolerance Coefficients (one-sided)

P = Percent of Population Values Below the Upper 95% Tolerance Limit

Sample Size (n)	Degrees of Freedom (DF)	P = 90% ($K_{.90}$)	P = 95% ($K_{.95}$)	P = 99% ($K_{.99}$)
2	1			
3	2	6.16	7.66	10.55
4	3	4.15	5.14	7.04
5	4	3.41	4.20	5.74
6	5	3.01	3.71	5.16
7	6	2.76	3.40	4.64
8	7	2.58	3.19	4.35
9	8	2.45	3.03	4.14
10	9	2.36	2.91	3.98
11	10	2.28	2.82	3.85
12	11	2.21	2.74	3.75
13	12	2.16	2.67	3.66
14	13	2.11	2.61	3.58
15	14	2.07	2.57	3.52
16	15	2.03	2.52	3.46
17	16	2.00	2.49	3.42
18	17	1.97	2.45	3.37
19	18	1.95	2.42	3.33
20	19	1.93	2.40	3.30
21	20	1.90	2.37	3.26
22	21	1.89	2.35	3.23
23	22	1.87	2.33	3.21
24	23	1.85	2.31	3.18
25	24	1.84	2.29	3.16
50	49	1.65	2.06	2.86
100	99	1.52	1.92	2.68
∞	∞	1.28	1.65	2.33

TABLE A-6

$$\textbf{SETUP FOR} \begin{cases} p \text{ transducers} \\ q \text{ consoles} \\ r \text{ repetitions} \end{cases}$$

CONSOLE
j = 1, 2, ..., q

TRANSDUCER
i
i = 1, 2, ... , p

	1	2	....	q	
1	m_{11}, s_{11}	m_{12}, s_{12}	....	m_{1q}, s_{1q}	$m_{i\cdot}$
2	m_{21}, s_{21}	m_{22}, s_{22}	....	m_{2q}, s_{2q}	$m_{2\cdot}$
.	.	.		.	.
.	.	.		.	.
.	.	.		.	.
p	m_{p1}, s_{p1}	m_{p2}, s_{p2}	....	m_{pq}, s_{pq}	$m_{p\cdot}$
	$m_{\cdot 1}$	$m_{\cdot 2}$	....	$m_{\cdot q}$	$\overline{\overline{m}}$

$S_{i\cdot}$

$S_{\cdot j}$

ijth cell mean $\qquad m_{ij} = \frac{1}{r}\sum_{k=1}^{r} x_{ijk}$

ith transducer mean $\qquad m_{i\cdot} = \frac{1}{q}\sum_{j=1}^{q} m_{ij}$

jth console mean $\qquad m_{\cdot j} = \frac{1}{p}\sum_{i-1}^{p} m_{ij}$

overall mean $\qquad \overline{\overline{m}} = \frac{1}{pq}\sum_{i=1}^{p}\sum_{j=1}^{q} m_{ij}$

variance of ijth cell $\qquad s_{ij}^2 = \sum_{k=1}^{r}\left(x_{ijk} - m_{ij}\right)^2 \Big/ (r-1)$

total variance of transducer $\qquad s_{i\cdot}^2 = \sum_{i=1}^{p}\left(m_{i\cdot} - \overline{\overline{m}}\right)^2 \Big/ (p-1)$

total variance of console $\qquad s_{\cdot j}^2 = \sum_{j=1}^{q}\left(m_{\cdot j} - \overline{\overline{m}}\right)^2 \Big/ (q-1)$

TABLE A-7

Power Measurements Of 12 Console-Transducer Combinations (milliwatts)

CONSOLE

J = 1, 2, 3, 4 (q = 4)

TRANSDUCER

i = 1, 2, 3 (p = 3)

Transducer	Console 1	Console 2	Console 3	Console 4		
1	64 77 72 56 68 95	55 64 66 70 49 68	63 71 66 59 53 72	73 66 55 64 72 78		
	MEAN = 72 STD DEV = 13.34	MEAN = 62 STD DEV = 8.22	MEAN = 64 STD DEV = 7.27	MEAN = 68 STD DEV = 8.12	$m_{1.} = 66.50$	
2	68 81 87 72 75 67	51 54 62 57 55 63	75 93 78 71 63 76	70 56 68 52 57 63		
	MEAN = 75 STD DEV = 7.77	MEAN = 57 STD DEV = 4.69	MEAN = 76 STD DEV = 9.88	MEAN = 61 STD DEV = 7.16	$m_{2.} = 67.25$	$S_{i.} = 10.18$
3	48 55 39 42 44 37	52 49 50 54 53 54	55 39 40 52 48 60	46 49 51 60 50 50		
	MEAN = 45 STD DEV = 9.88	MEAN = 52 STD DEV = 2.10	MEAN = 49 STD DEV = 8.34	MEAN = 51 STD DEV = 4.73	$m_{3.} = 49.25$	
	$m_{.1} = 64.00$	$m_{.2} = 57.00$	$m_{.3} = 63.00$	$m_{.4} = 60.00$	$\bar{\bar{m}} = 61.00$	

$S_{.j} = 3.16$

Overall mean = 61.00 mW

Power = 61 ± 14 mW (95% C.I.)

Power ≤ 98 mW (95% T.I.)

REFERENCES

Ziskin MC: Measurement uncertainty in ultrasonic exposimetry. Chapter 14 in *Ultrasonic Exposimetry* edited by Ziskin MC and Lewin PA. CRC Press, Boca Raton, pp. 409–443, 1993.

Harris IA and Hinton LJT. *The Expression of Uncertainty in Electrical Measurements.* British Calibration Service publication No. 3003, 1984.

BIBLIOGRAPHY

Bevington PR: *Data Reduction and Error Analysis for the Physical Sciences.* McGraw-Hill, New York, 1969.

Ludwig G and Brendel K: Calibration of Hydrophones Based on Reciprocity and Time Delay Spectrometry. *IEEE Trans. on Ultrasonics, Ferroelectrics, and Frequency Control* 35:168–174, 1988.

Meyer SL: *Data Analysis for Scientists and Engineers*. John Wiley, New York, 1975.

National Council on Radiation Protection and Measurements: *A Handbook of Radioactivity Measurements Procedures.* NCRP Report No. 58, Washington, DC, 1978.

Natrella MG: *Experimental Statistics. National Bureau of Standards Handbook* 91, Washington, DC, 1963.

Parker KJ and Lyons ME: Absorption and Attenuation in Soft Tissues: I-Calibration and Error Analyses. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control,* 35:242–252, 1988.

Preston RC, Bacon DR and Smith RA: Calibration of Medical Ultrasonic Equipment and Accuracy Assessment. *IEEE Transactions on Ultrasonics, Ferroelectrics, and Frequency Control,* 35:110–121, 1988.

Snedecor GW and Cochran WG: *Statistical Methods.* Seventh Edition. The Iowa State University Press, Ames, IA, 1980.